Light Propagation Volumes - Corrections

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1 Notation

$$\overline{f(x)} := max\{f(x), 0\}$$

$$\textit{flux } \Phi \textit{ of a light (energy / time)}$$

$$\textit{radiant intensity } I := \frac{d\Phi}{d\omega}, \textit{ with direction } \omega$$

$$\Phi = \int_{S^2} I(\omega)\ d\omega$$

$$\textit{radiance } L := \frac{d^2\Phi}{dAd\omega}, \textit{ with area } A$$

See Real-Time Rendering or Physically Based Rendering for more details. The notation tries to stick to Real-Time Rendering.

2 Light Injection

2.1 Intensity Formula Correction

Crytek's LPV paper says that, if $I_p(\omega)$ is the radiant intensity of an VPL and n_p is its normal and Φ_p the reflected flux, then:

$$I_p(\omega) = \Phi_p \overline{\langle n_p, \omega \rangle}$$

However:

$$\Phi_p = \int_{S^2} I_p(\omega) d\omega$$

If we expand the equation above:

$$\Phi_p = \int_{S^2} \Phi_p \overline{\langle n_p, \omega \rangle} d\omega = \Phi_p \int_{\Omega} \langle n_p, \omega \rangle\ d\omega = \Phi_p \cdot \pi \neq \Phi_p$$

I suggest the following correction factor:

$$I_p(\omega) = \frac{\Phi_p}{\pi} \overline{\langle n_p, \omega \rangle}$$

That is, the original equation is divided by π as normalization factor.

3 Propagation

3.1 Intensity Propagation Correction

Crytek's LPV paper uses the following formula for computing the flux to a neighbouring face:

$$\Phi_f = \int_\Omega I(\omega)V(\omega)\,d\omega$$

and approximates it using the solid angle $\Delta\omega_f = \int_\Omega V(\omega)\,d\omega$ and the central direction ω_c as

$$\Phi_f = \frac{\Delta\omega_f}{4\pi} \cdot I(\omega_c)$$

If we insert the definition of $\Delta\omega_f$ back into the formula, we get:

$$\Phi_f = \frac{1}{4\pi} I(\omega_c) \int_\Omega V(\omega)\,d\omega = \frac{1}{4\pi} \int_\Omega I(\omega_c)V(\omega)\,d\omega$$

If we assume that $I(\omega_c)$ is constant, we immediately see that the division by 4π does not make sense here.

3.2 Reprojection Formula Correction

Crytek's LPV paper uses the formulas:

$$\Phi_f = \int_\Omega \Phi_l \overline{\langle n_l, \omega\rangle}\,d\omega$$

and

$$\Phi_l = \Phi_f / \pi$$

The integrand is exactly the intensity that had to be corrected in 2.1. So now we get:

$$\Phi_f = \int_\Omega \frac{1}{\pi}\Phi_l \overline{\langle n_l, \omega\rangle}\,d\omega = \frac{1}{\pi}\Phi_l \int_\Omega \overline{\langle n_l, \omega\rangle}\,d\omega = \frac{1}{\pi}\Phi_l\,\pi = \Phi_l$$

This also makes more sense because now energy conservation is obeyed during reprojection: now the whole flux that arrives at the face of the neighbouring cell is reprojected into this cell and nothing is lost. However, because of the intensity correction, the net value stays the same. This is just a correction to make the intermediate values physically more correct.